Optimum number of azimuth angles for an airborne, step-stare scanning, coherent wind lidar

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October 16, 2012

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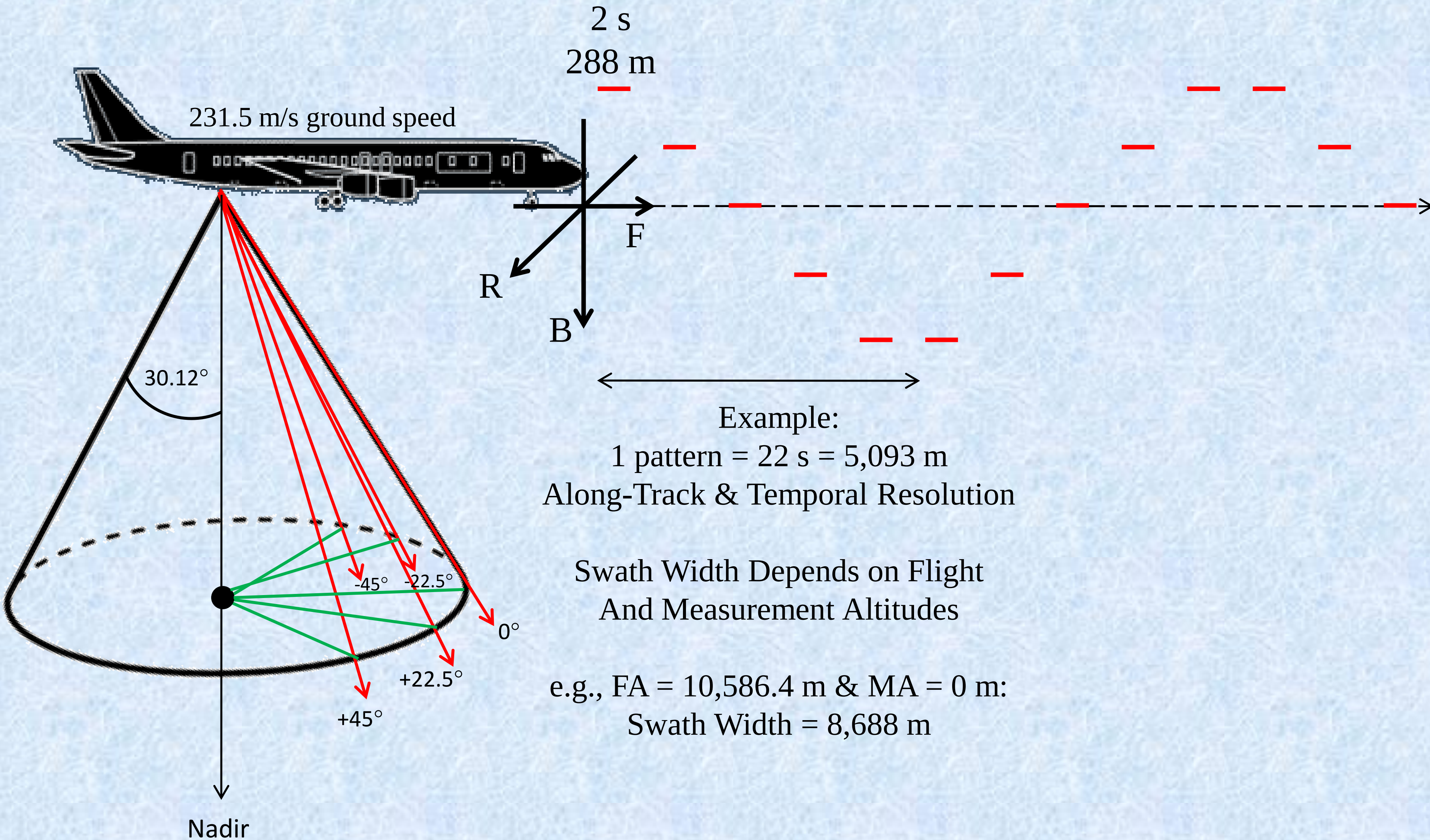
Nominal Scan Pattern: DAWN During GRIP Campaign

5 different azimuth angles from -45 to + 45°
2 sec shot integration; 2 sec scanner turn time



Open Questions

1. Is it better to have equal-spaced azimuth angles or not?
2. How many azimuth angles is optimum?
3. Does it depend on the wind field?
4. Does it depend on the desired science products?
5. Does it depend on whether the wind field is assumed horizontally homogeneous in the measurement volume?
6. Does it depend on the aircraft velocity?
7. Does it depend on the scanner change time?
8. Should the angles be probed sequentially or in a different order?

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Only a start on 3 of the questions …

Past Numbers

- DAWN, DC-8, LaRC: <u>5</u> irregular azimuths, step-stare conical
- TODWL, P3DWL, SWA: <u>12</u> evenly spaced azimuths, step-stare conical
- WIND, DLR: <u>200</u> evenly space azimuths, continuous conical, not exactly repeated

Setting Up the Problem

N_{SH} – laser shots per azimuth angle [-]

f_{LA} – laser PRF [Hz]

t_{CH} – time to change azimuth angles [s]

N_{AZ} – azimuth angles per shot pattern [-]

V_{AC} – aircraft horizontal velocity [m/s]

L_{SCAN} – scan pattern horizontal resolution

Lidar Shot Spacing = $F_{SHOT} = \frac{V_{AC}}{f_{LA}}$

Time of azimuth angle (LOS) = $t_{AZ} = \frac{N_{SH}}{f_{LA}} + t_{CH}$

Length of azimuth angle $L_{AZ} = t_{AZ}V_{AC} = V_{AC}\left(\frac{N_{SH}}{f_{LA}} + t_{CH}\right) = d_{MIN}N_{SH} + V_{AC}t_{CH}$

Time of scan pattern $t_{SCAN} = N_{AZ}t_{AZ} = N_{AZ}\left(\frac{N_{SH}}{f_{LA}} + t_{CH}\right)$

Horizontal resolution $L_{SCAN} = V_{AC}t_{SCAN} = V_{AC}N_{AZ}t_{AZ} = N_{AZ}L_{AZ} = N_{AZ}V_{AC}\left(\frac{N_{SH}}{f_{LA}} + t_{CH}\right)\,[m] \quad \therefore \quad L_{AZ} = \frac{L_{SCAN}}{N_{AZ}}$

$$\therefore\ N_{SH} = f_{LA}\left[\frac{L_{SCAN}}{V_{AC}N_{AZ}} - t_{CH}\right] = \frac{f_{LA}}{V_{AC}N_{AZ}}\left[L_{SCAN} - t_{CH}V_{AC}N_{AZ}\right] \quad \therefore \quad N_{AZ} = \frac{L_{SCAN}\,f_{LA}}{V_{AC}\left(N_{SH} + f_{LA}t_{CH}\right)}$$

Setting Up the Figure of Merit to Minimize

1. Velocity error variance of one azimuth angle* $\sigma_{AZ}^2 = \frac{C_{AZ}^2}{N_{SH}} L_{AZ}^{2/3} = \frac{C_{AZ}^2}{N_{SH}} \left(\frac{L_{SCAN}}{N_{AZ}} \right)^{2/3}$ $C_{AZ}^2 \left[\frac{m^2}{s^2 m^{2/3}} = \frac{m^{4/3}}{s^2} \right]$

[there actually are 2 coherent wind lidar FOMS: σ_G and Pr{good}.
Assume Pr{good} close enough to 1 to ignore.]

2. Velocity error variance from one scan pattern, from combining many LOS wind values $\sigma_{SCAN}^2 = \frac{C_{SCAN}^2}{N_{AZ}}$ $C_{SCAN}^2 \left[\frac{m^2}{s^2} \right]$

3. Velocity error variance from cloud/rain blockage $\downarrow \sigma_{HOLE}^2 = \frac{C_{HOLE}^2}{N_{AZ}^2}$ $C_{HOLE}^2 \left[\frac{m^2}{s^2} \right]$

Define FOM $\triangleq W_{AZ} \left(\sigma_{AZ} \right)^2 + W_{SCAN} \left(\sigma_{SCAN} \right)^2 + W_{HOLE} \left(\sigma_{HOLE} \right)^2$ $\left[\frac{m^2}{s^2} \right]$ $W[-]$

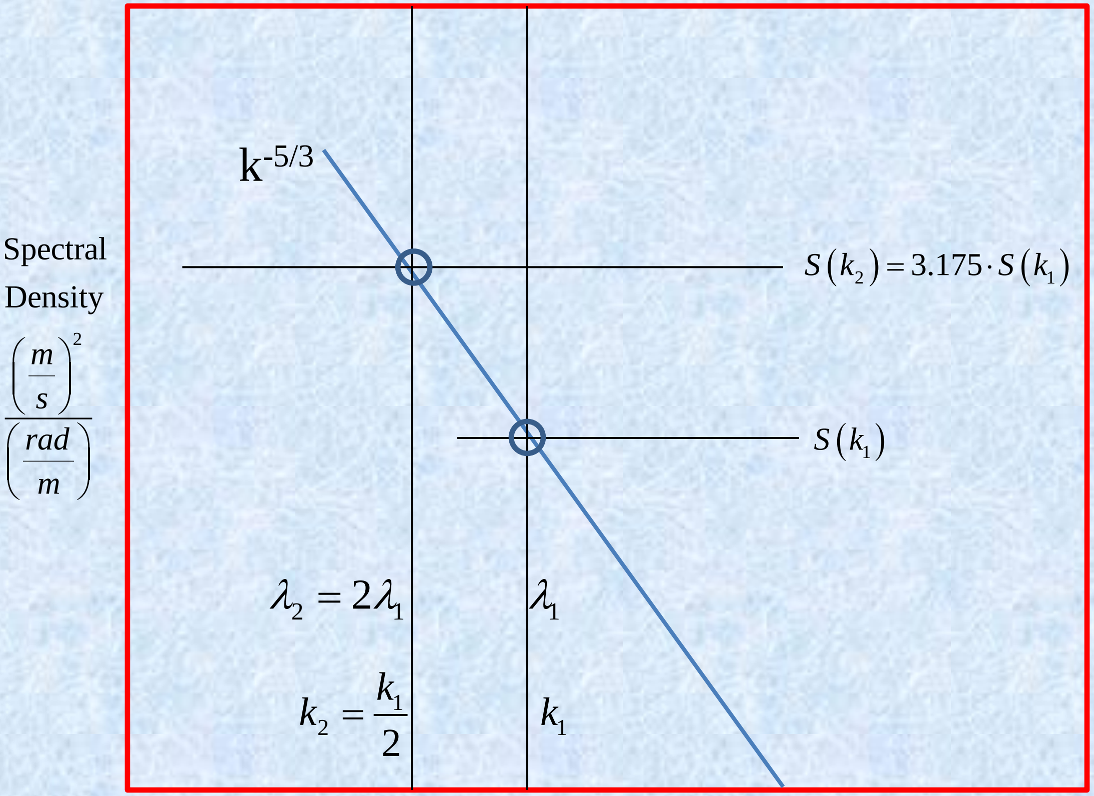
**The exponent 1/3 (standard deviation) for distance is from Yahaya and Frangi, "Profile of the horizontal wind variance near the ground in near neutral flow – K-theory and the transport of the turbulent kinetic energy," Ann. Geophys. 27, 1843-1859 (2009), Eq. (1)*

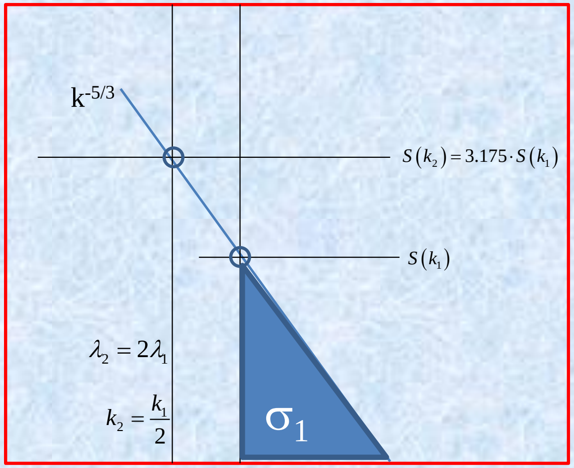
Derivation of $L_{AZ}^{1/3}$

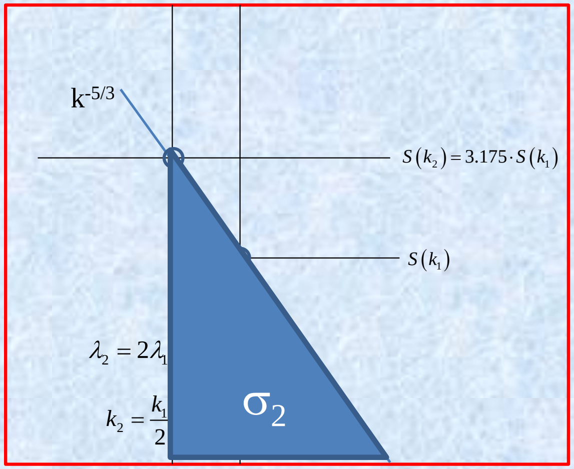
$$k = \frac{2\pi}{\lambda}$$

$$S(k_1) = Ck_1^{-5/3}$$

$$k_1 = \frac{2\pi}{\lambda_1} \qquad k_2 = \frac{2\pi}{\lambda_2} = \frac{2\pi}{2\lambda_1} = \frac{k_1}{2} \qquad S(k_2) = Ck_2^{-5/3} = C\left(\frac{k_1}{2}\right)^{-5/3} = 2^{5/3} S(k_1) = 3.175 \cdot S(k_1)$$







0

$$\int_k^\infty S(s)ds = C\int_k^\infty s^{-5/3}ds = C\left[-\frac{3}{2}s^{-2/3}\right]_k^\infty = -C\frac{3}{2}\left[0 - k^{-2/3}\right] = C\frac{3}{2}k^{-2/3} = C\frac{3}{2k^{2/3}} = C\frac{3\lambda^{2/3}}{2(2\pi)^{2/3}} = C \cdot 0.44\lambda^{2/3}$$

$$\sigma(\lambda) \propto \sqrt{\int_k^\infty S(s)ds} = \sqrt{C \cdot 0.44\lambda^{2/3}} = 0.66\sqrt{C}\,\lambda^{1/3} \qquad \sigma^2(2\lambda) = C \cdot 0.44(2\lambda)^{2/3} = 1.59\,\sigma^2(\lambda) = +59\%$$

Expand FOM
and
Eliminate Variable N_{SH} from Equation by Using L_{SCAN} Equation

$$\text{FOM} \triangleq W_{AZ}\left(\sigma_{AZ}\right)^2 + W_{SCAN}\left(\sigma_{SCAN}\right)^2 + W_{HOLE}\left(\sigma_{HOLE}\right)^2 \qquad \left[\frac{m^2}{s^2}\right]$$

$$= W_{AZ}\frac{C_{AZ}^2}{N_{SH}}\left(\frac{\Delta h}{N_{AZ}}\right)^{2/3} + \frac{W_{SCAN}C_{SCAN}^2}{N_{AZ}} + \frac{W_{HOLE}C_{HOLE}^2}{N_{AZ}^2} = W_{AZ}\frac{C_{AZ}^2}{\left[\frac{f_{LA}}{V_{AC}N_{AZ}}\left[L_{SCAN} - t_{CH}V_{AC}N_{AZ}\right]\right]}\left(\frac{L_{SCAN}}{N_{AZ}}\right)^{2/3} + \frac{W_{SCAN}C_{SCAN}^2}{N_{AZ}} + \frac{W_{HOLE}C_{HOLE}^2}{N_{AZ}^2}$$

$$= W_{AZ}\frac{C_{AZ}^2 V_{AC} N_{AZ}}{\left[f_{LA}\left[L_{SCAN} - t_{CH}V_{AC}N_{AZ}\right]\right]}\left(\frac{L_{SCAN}}{N_{AZ}}\right)^{2/3} + \frac{W_{SCAN}C_{SCAN}^2}{N_{AZ}} + \frac{W_{HOLE}C_{HOLE}^2}{N_{AZ}^2} = \frac{W_{AZ}C_{AZ}^2 V_{AC} L_{SCAN}^{2/3} N_{AZ}^{1/3}}{f_{LA}\left[L_{SCAN} - t_{CH}V_{AC}N_{AZ}\right]} + \frac{W_{SCAN}C_{SCAN}^2}{N_{AZ}} + \frac{W_{HOLE}C_{HOLE}^2}{N_{AZ}^2}$$

$$FOM = \frac{W_{AZ}C_{AZ}^2 V_{AC} L_{SCAN}^{2/3} N_{AZ}^{1/3}}{f_{LA}\left[L_{SCAN} - t_{CH}V_{AC}N_{AZ}\right]} + \frac{W_{SCAN}C_{SCAN}^2}{N_{AZ}} + \frac{W_{HOLE}C_{HOLE}^2}{N_{AZ}^2}$$

One silly minimum occurs for $N_{AZ} \rightarrow \infty$

Find Optimum Value of N_{AZ} by Differentiating and Setting to Zero

$$\frac{\partial FOM}{\partial N_{AZ}} = \frac{\partial}{\partial N_{AZ}}\left[\frac{W_{AZ}C_{AZ}^2V_{AC}L_{SCAN}^{2/3}N_{AZ}^{1/3}}{\left[f_{LA}L_{SCAN} - f_{LA}t_{CH}V_{AC}N_{AZ}\right]} + \frac{W_{SCAN}C_{SCAN}^2}{N_{AZ}} + \frac{W_{HOLE}C_{HOLE}^2}{N_{AZ}^2}\right] = 0 \qquad \left[\frac{m^2}{s^2}\right]$$

$$\frac{\left[f_{LA}L_{SCAN} - f_{LA}t_{CH}V_{AC}N_{AZ}\right]\left[\frac{1}{3}W_{AZ}C_{AZ}^2V_{AC}L_{SCAN}^{2/3}N_{AZ}^{-2/3}\right] - \left[W_{AZ}C_{AZ}^2V_{AC}L_{SCAN}^{2/3}N_{AZ}^{1/3}\right]\left[0 - f_{LA}t_{CH}V_{AC}\right]}{\left[f_{LA}L_{SCAN} - f_{LA}t_{CH}V_{AC}N_{AZ}\right]^2} - \frac{W_{PA}C_{PA}^2}{N_{AZ}^2} - \frac{2W_{HOLE}C_{HOLE}^2}{N_{AZ}^3} = 0$$

$$\frac{\left[f_{LA}\frac{1}{3}W_{AZ}C_{AZ}^2V_{AC}L_{SCAN}^{5/3}N_{AZ}^{-2/3} + \frac{2}{3}f_{LA}t_{CH}V_{AC}^2W_{AZ}C_{AZ}^2L_{SCAN}^{2/3}N_{AZ}^{1/3}\right]}{\left[f_{LA}L_{SCAN} - f_{LA}t_{CH}V_{AC}N_{AZ}\right]^2} - \frac{W_{PA}C_{PA}^2}{N_{AZ}^2} - \frac{2W_{HOLE}C_{HOLE}^2}{N_{AZ}^3} = 0$$

$$\frac{\left[f_{LA}\frac{1}{3}W_{AZ}C_{AZ}^2V_{AC}L_{SCAN}^{5/3}N_{AZ}^{+7/3} + \frac{2}{3}f_{LA}t_{CH}V_{AC}^2W_{AZ}C_{AZ}^2L_{SCAN}^{2/3}N_{AZ}^{10/3}\right]}{\left[f_{LA}L_{SCAN} - f_{LA}t_{CH}V_{AC}N_{AZ}\right]^2} - W_{PA}C_{PA}^2N_{AZ} - 2W_{HOLE}C_{HOLE}^2 = 0$$

$$\left[f_{LA}\frac{1}{3}W_{AZ}C_{AZ}^2V_{AC}L_{SCAN}^{5/3}N_{AZ}^{+7/3} + \frac{2}{3}f_{LA}t_{CH}V_{AC}^2W_{AZ}C_{AZ}^2L_{SCAN}^{2/3}N_{AZ}^{10/3}\right]$$
$$-W_{PA}C_{PA}^2\left[f_{LA}^2L_{SCAN}^2N_{AZ} - 2f_{LA}^2L_{SCAN}t_{CH}V_{AC}N_{AZ}^2 + f_{LA}^2t_{CH}^2V_{AC}^2N_{AZ}^3\right] - 2W_{HOLE}C_{HOLE}^2\left[f_{LA}^2L_{SCAN}^2 - 2f_{LA}^2L_{SCAN}t_{CH}V_{AC}N_{AZ} + f_{LA}^2t_{CH}^2V_{AC}^2N_{AZ}^2\right] = 0 \qquad \left[\frac{m^4}{s^4}\right]$$

$$f_{LA}\frac{1}{3}W_{AZ}C_{AZ}^2V_{AC}L_{SCAN}^{5/3}N_{AZ}^{+7/3} + \frac{2}{3}f_{LA}t_{CH}V_{AC}^2W_{AZ}C_{AZ}^2L_{SCAN}^{2/3}N_{AZ}^{10/3} - W_{PA}C_{PA}^2f_{LA}^2t_{CH}^2V_{AC}^2N_{AZ}^3 + \left(W_{PA}C_{PA}^2\,2f_{LA}^2L_{SCAN}t_{CH}V_{AC} - 2W_{HOLE}C_{HOLE}^2f_{LA}^2t_{CH}^2V_{AC}^2\right)N_{AZ}^2$$
$$+\left(2W_{HOLE}C_{HOLE}^2\,2f_{LA}^2L_{SCAN}t_{CH}V_{AC} - W_{PA}C_{PA}^2f_{LA}^2L_{SCAN}^2\right)N_{AZ} - 2W_{HOLE}C_{HOLE}^2f_{LA}^2L_{SCAN}^2 = 0$$

Find Optimum Value of N_{AZ} by Differentiating and Setting to Zero

$$\frac{2}{3} t_{CH} V_{AC}^2 W_{AZ} C_{AZ}^2 L_{SCAN}^{2/3} N_{AZ}^{10/3} - W_{SCAN} C_{SCAN}^2 f_{LA} t_{CH}^2 V_{AC}^2 N_{AZ}^3 + \frac{1}{3} W_{AZ} C_{AZ}^2 V_{AC} L_{SCAN}^{5/3} N_{AZ}^{7/3} + \left(W_{SCAN} C_{SCAN}^2 2 f_{LA} L_{SCAN} t_{CH} V_{AC} - 2 W_{HOLE} C_{HOLE}^2 f_{LA} t_{CH}^2 V_{AC}^2 \right) N_{AZ}^2$$
$$+ \left(4 W_{HOLE} C_{HOLE}^2 f_{LA} L_{SCAN} t_{CH} V_{AC} - W_{SCAN} C_{SCAN}^2 f_{LA} L_{SCAN}^2 \right) N_{AZ} - 2 W_{HOLE} C_{HOLE}^2 f_{LA} L_{SCAN}^2 = 0$$

	$N_{AZ}^{10/3}$	N_{AZ}^{3}	$N_{AZ}^{7/3}$	N_{AZ}^{2}	N_{AZ}	N_{AZ}^{0}
W_{AZ}	+		+			
W_{SCAN}		-		+	-	
W_{HOLE}				-	+	-

Divide by W_{AZ} to obtain ratios of weights:

$$\frac{1}{3} C_{AZ}^2 V_{AC} \Delta h^{5/3} N_{AZ}^{+7/3} + \frac{2}{3} t_{CH} V_{AC}^2 C_{AZ}^2 \Delta h^{2/3} N_{AZ}^{10/3} - \frac{W_{PA}}{W_{AZ}} C_{PA}^2 f_{LA} t_{CH}^2 V_{AC}^2 N_{AZ}^3 + \left(\frac{W_{PA}}{W_{AZ}} C_{PA}^2 2 f_{LA} \Delta h t_{CH} V_{AC} - 2 \frac{W_{HOLE}}{W_{AZ}} C_{HOLE}^2 f_{LA} t_{CH}^2 V_{AC}^2 \right) N_{AZ}^2$$
$$+ \left(4 \frac{W_{HOLE}}{W_{AZ}} C_{HOLE}^2 f_{LA} \Delta h t_{CH} V_{AC} - \frac{W_{PA}}{W_{AZ}} C_{PA}^2 f_{LA} \Delta h^2 \right) N_{AZ} - 2 \frac{W_{HOLE}}{W_{AZ}} C_{HOLE}^2 f_{LA} \Delta h^2 = 0$$

Algebraic mess, so
Use MS Excel to find the zero crossings of this equation

Choose Values and Approximate Values for the Excel Calculation

$f_{LA} = 10\ s^{-1}$ $\quad N_{SH} = 20$ $\quad t_{CH} = 2\ s$

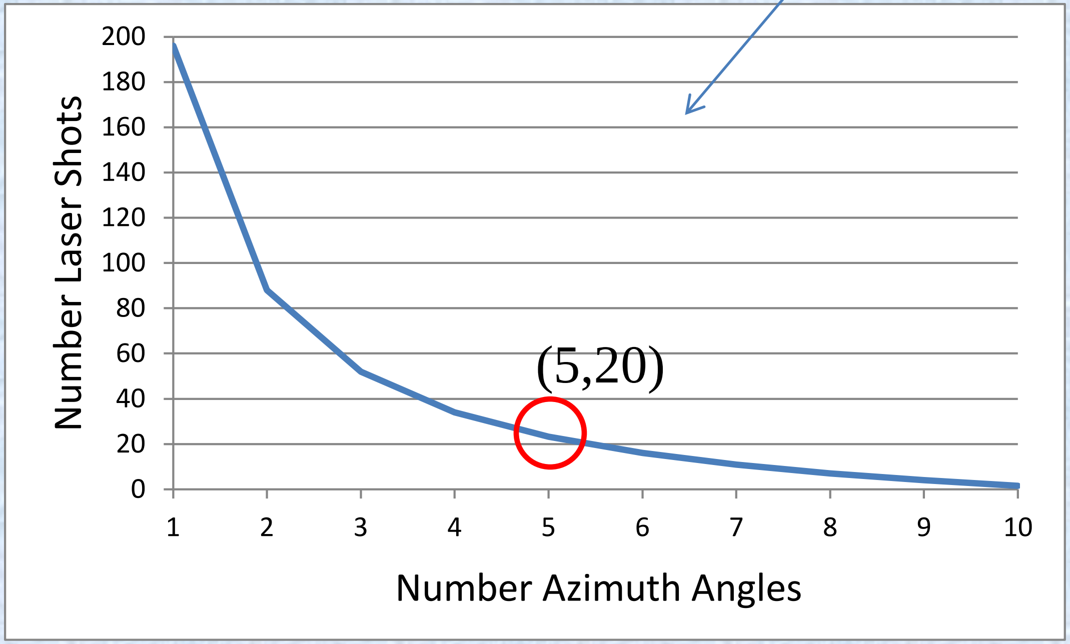
$t_{AZ} = 4\ s$ $\quad V_{AC,DC-8} = 231.5\ m/s$ $\quad V_{AC,UC-12B} = 133.8\ m/s$

$C_{AZ} \sim 2\ m/s$ $\quad C_{SCAN} \sim 3\ m/s$ $\quad C_{HOLE} \sim 4\ m/s$

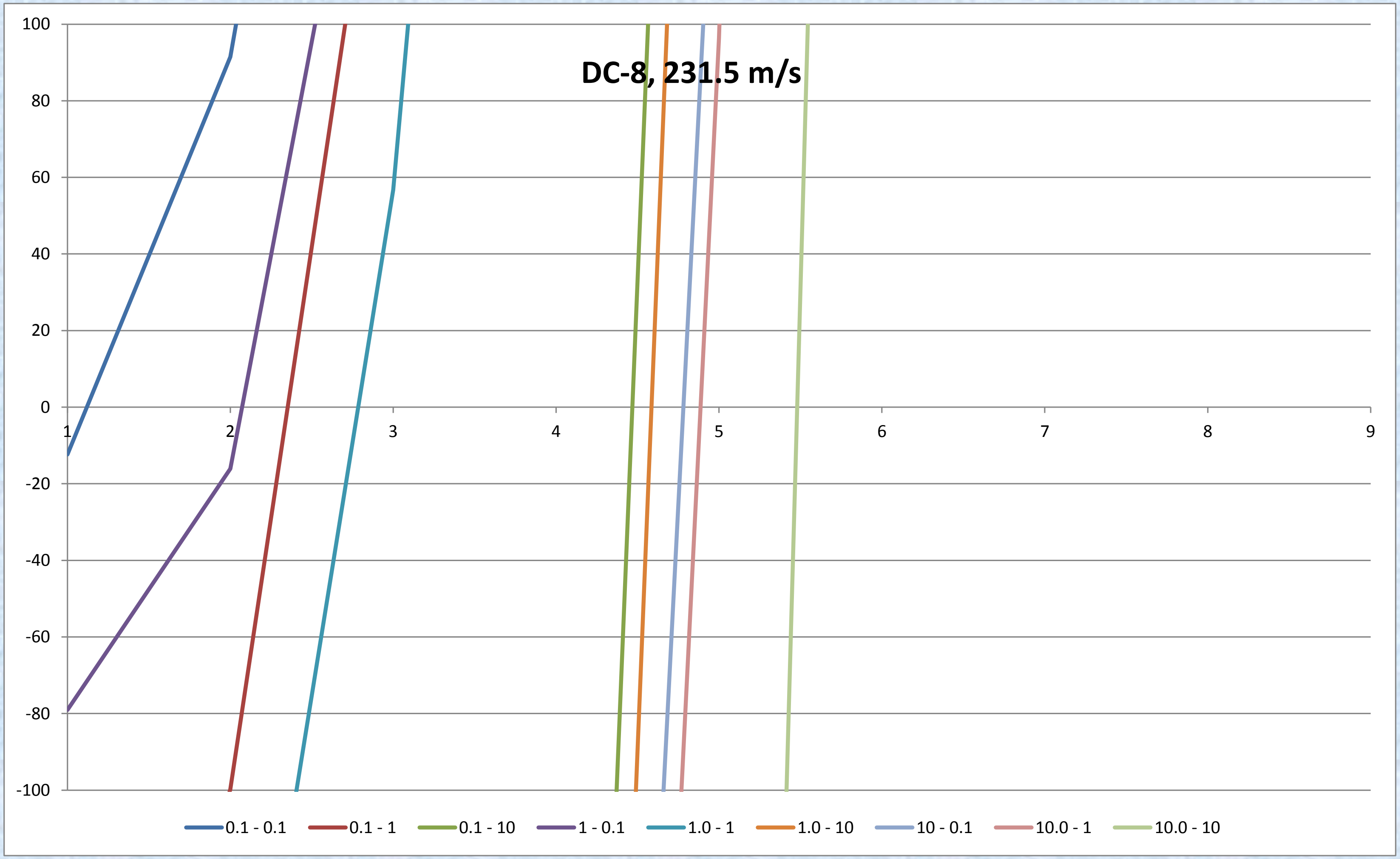
$L_{SCAN} \sim 5,000\ m$

$$\therefore N_{SH} = f_{LA}\left[\frac{L_{SCAN}}{V_{AC}N_{AZ}} - t_{CH}\right] = 10\left[\frac{5000}{231.5N_{AZ}} - 2\right] = \frac{216}{N_{AZ}} - 20$$

For these assumed parameter values & DC-8:



Results for DC-8



The zero crossings are the optimum number of azimuth angles

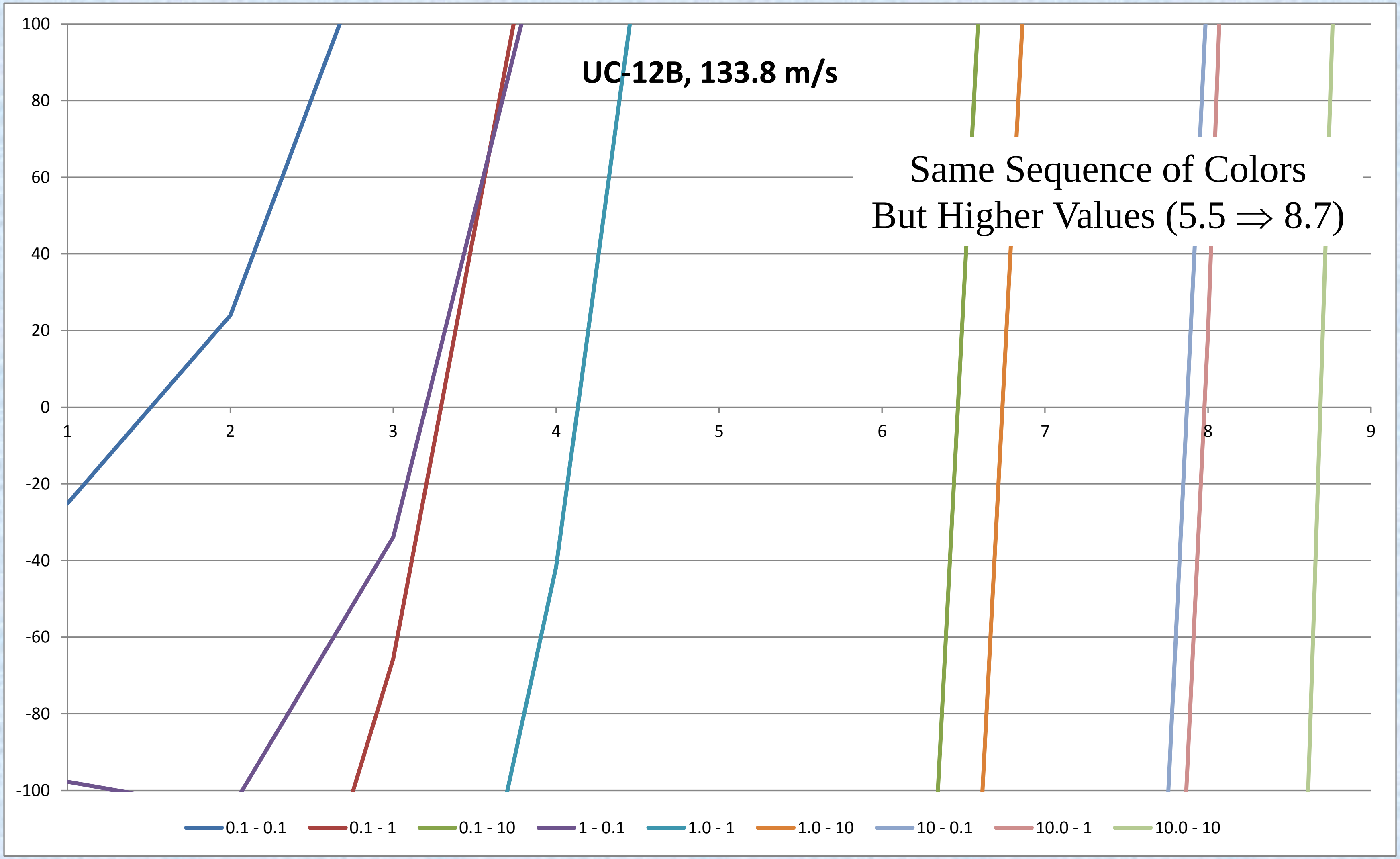
Legend colors are labeled by: (relative importance ratio of lowering scan pattern to single azimuth error – relative importance ratio of lowering cloud/rain blockage to single azimuth error):

$$\left(\frac{W_{SCAN}}{W_{AZ}} - \frac{W_{HOLE}}{W_{AZ}} \right)$$

The highest optimum azimuth number (5.5) is for 10 – 10

The lowest optimum azimuth number (1.2) is for 0.1 – 0.1

Results for UC-12B



The zero crossings are the optimum number of azimuth angles.
Legend colors are labeled by: (relative importance ratio of lowering scan pattern to single azimuth error – relative importance ratio of lowering cloud/rain blockage to single azimuth error):

$$\left(\frac{W_{SCAN}}{W_{AZ}} - \frac{W_{HOLE}}{W_{AZ}} \right)$$

The highest optimum azimuth number (8.7) is for 10 – 10
The lowest optimum azimuth number (1.5) is for 0.1 – 0.1

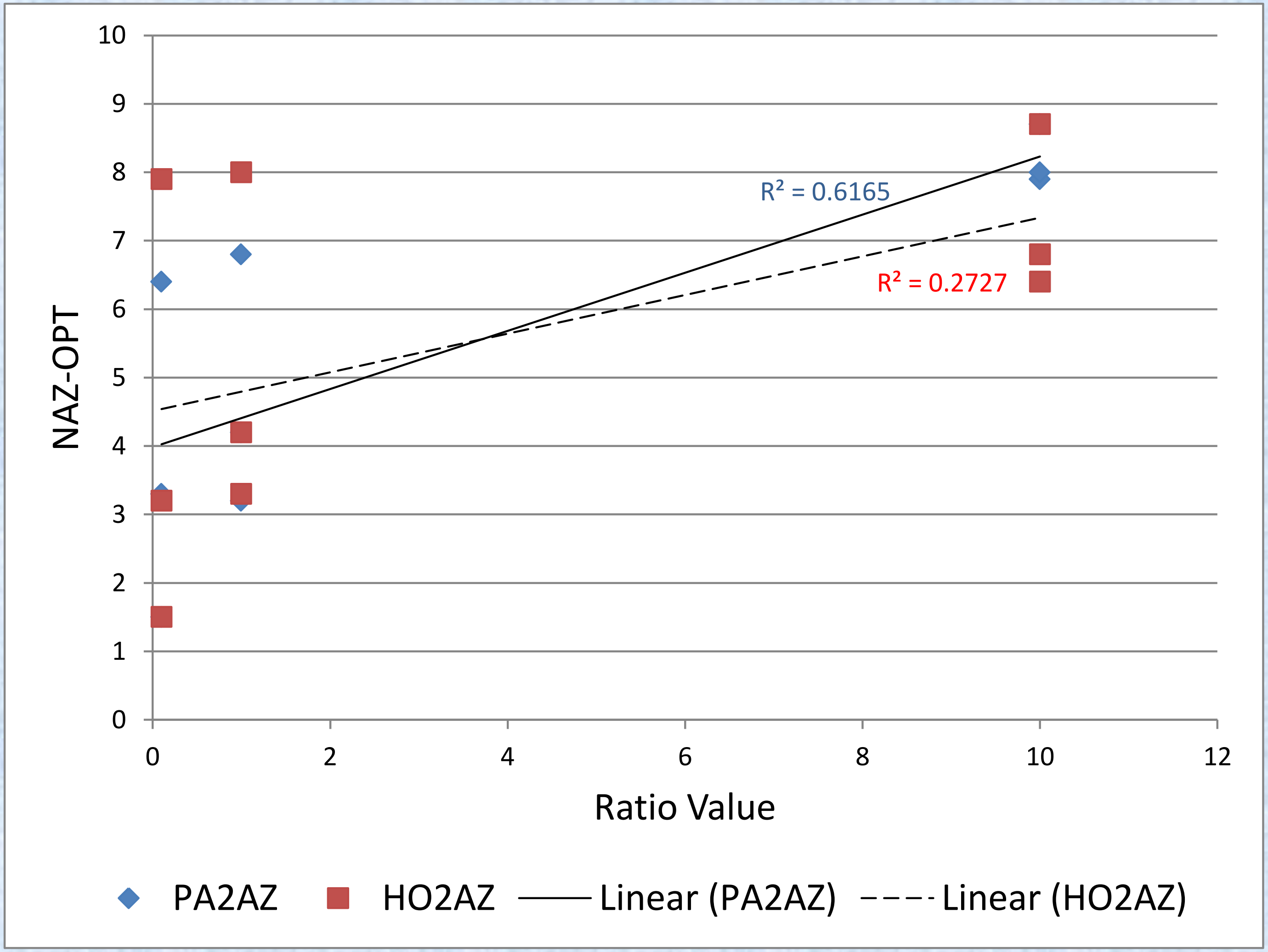
Sequence of Ascending Optimum

$\frac{W_{SCAN}}{W_{AZ}}$	0.1	1	0.1	1	0.1	1	10	10	10
$\frac{W_{HOLE}}{W_{AZ}}$	0.1	0.1	1	1	10	10	0.1	1	10
RMS	0.1	0.7	0.7	1	7	7	7	7	10

Optimum Number of Azimuth Angles →

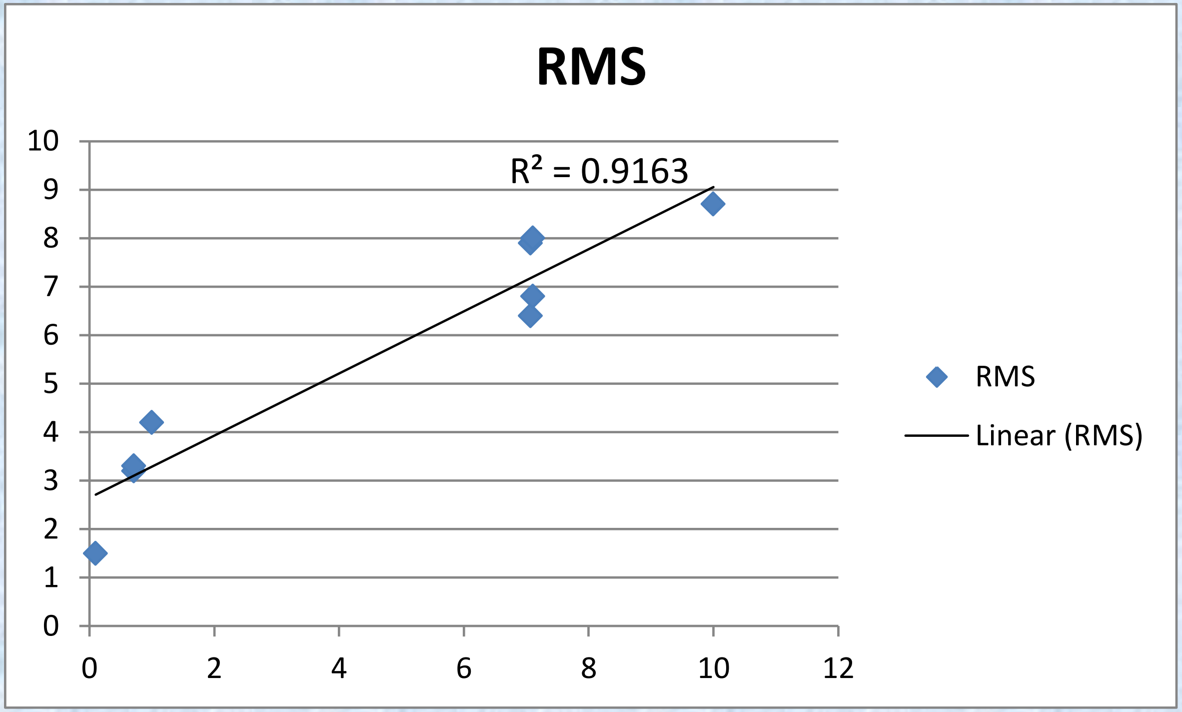
Which ratio gives the best hint of optimum number azimuths?

Sequence of Colors
Ratio Values vs. N_{AZ}^{OPT} Scatter Plot
Linear Fit R^2 Correlation Comparison



$$\frac{W_{SCAN}}{W_{AZ}} \text{ is ratio with best } R^2 \text{ value}$$

RMS is even better: $$\sqrt{\frac{1}{2}\left[\left(\frac{W_{SCAN}}{W_{AZ}}\right)^2+\left(\frac{W_{HOLE}}{W_{AZ}}\right)^2\right]}$$



- RMS or
- quadratic mean

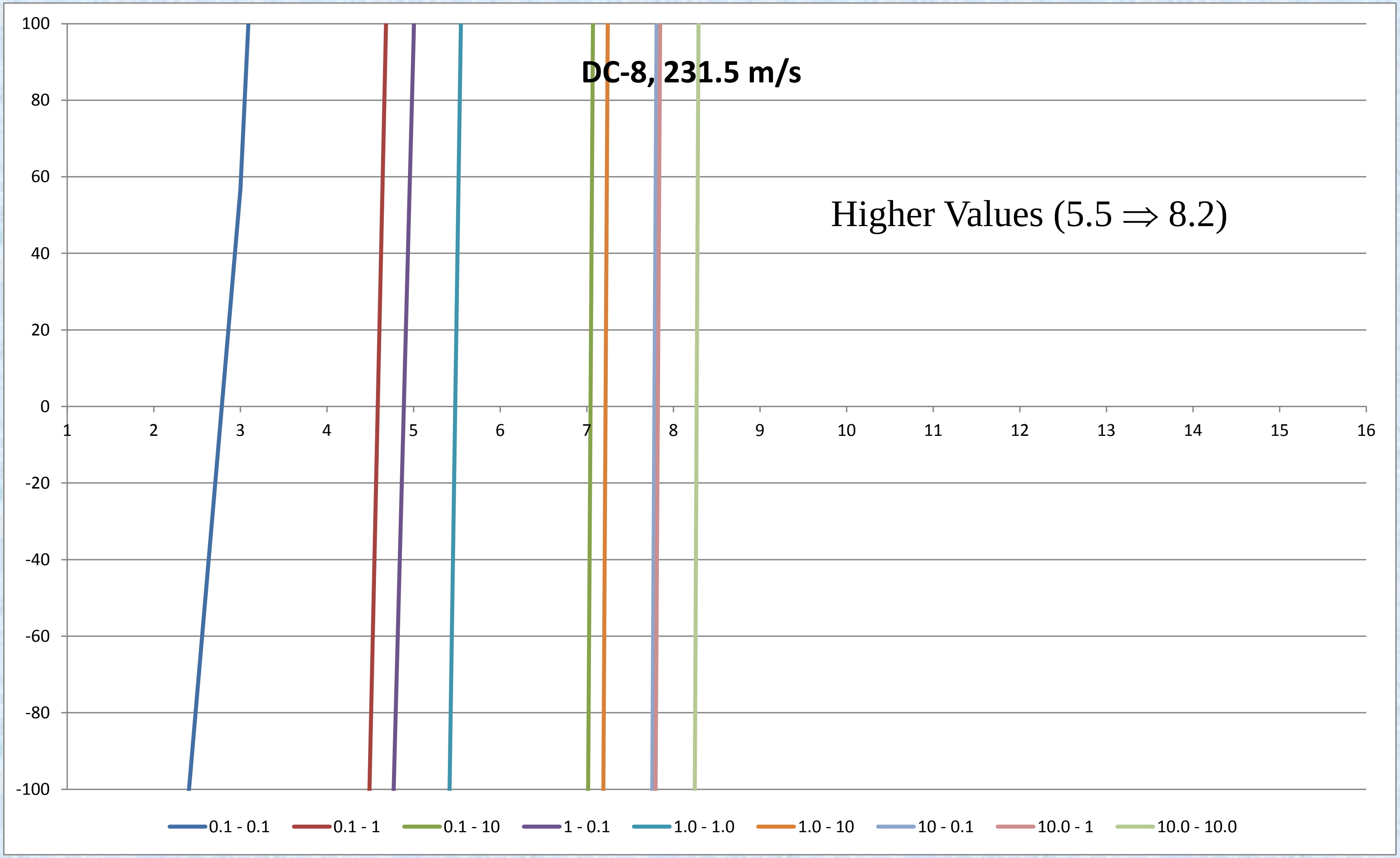
Special case of:

- generalized mean or
- power mean or
- Hölder mean

- As the importance ratio of combining many LOS winds, to measuring one LOS wind, goes higher; the optimum number of azimuth angles increases
- Airplane speed matters
- Regardless of this calculation, the number of azimuths should never be less than the number of unknown values that are needed from using land returns

Laser Pulse Rate 10x Faster

$$f_{LA} \rightarrow 100\ s^{-1}$$

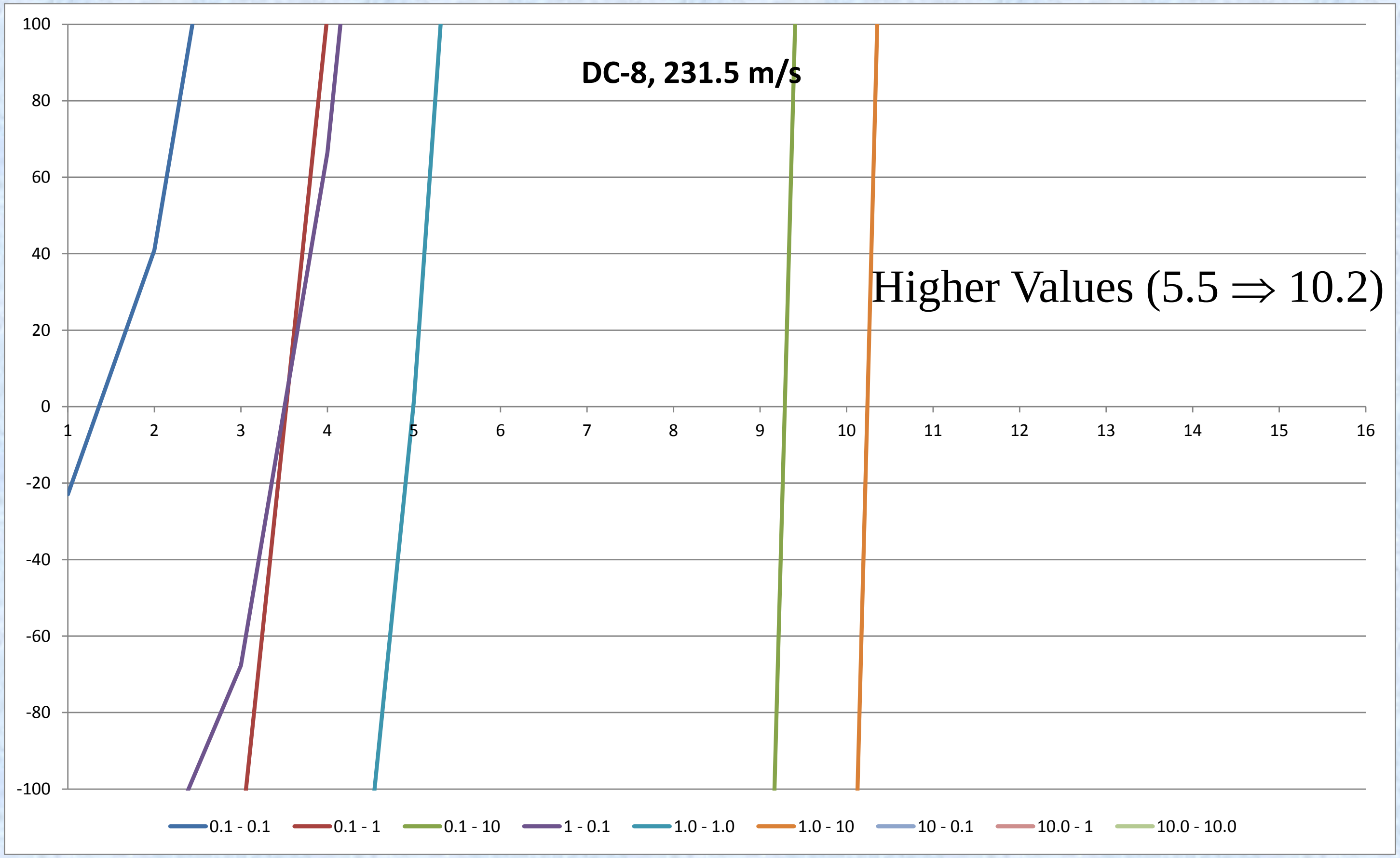


Infinitely Fast Azimuth Change Time

$$\frac{1}{3}C_{AZ}^2 V_{AC}\Delta h^{5/3}N_{AZ}^{+7/3}+\frac{2}{3}t_{CH}V_{AC}^2 C_{AZ}^2\Delta h^{2/3}N_{AZ}^{10/3}-\frac{W_{PA}}{W_{AZ}}C_{PA}^2 f_{LA}t_{CH}^2 V_{AC}^2 N_{AZ}^3+\left(\frac{W_{PA}}{W_{AZ}}C_{PA}^2 2f_{LA}\Delta h t_{CH}V_{AC}-2\frac{W_{HOLE}}{W_{AZ}}C_{HOLE}^2 f_{LA}t_{CH}^2 V_{AC}^2\right)N_{AZ}^2$$

$$+\left(4\frac{W_{HOLE}}{W_{AZ}}C_{HOLE}^2 f_{LA}\Delta h t_{CH}V_{AC}-\frac{W_{PA}}{W_{AZ}}C_{PA}^2 f_{LA}\Delta h^2\right)N_{AZ}-2\frac{W_{HOLE}}{W_{AZ}}C_{HOLE}^2 f_{LA}\Delta h^2=0$$

$$Let\ t_{CH}\rightarrow 0:\quad \frac{1}{3}C_{AZ}^2 V_{AC}\Delta h^{-1/3}N_{AZ}^{+7/3}-\frac{W_{PA}}{W_{AZ}}C_{PA}^2 f_{LA}N_{AZ}-2\frac{W_{HOLE}}{W_{AZ}}C_{HOLE}^2 f_{LA}=0$$



Conclusions

- The issue is important for having good airborne wind results when assimilating into models
- And that is good for generating advocacy for a space mission
- The list of questions at beginning is only partially addressed here
- Request all comments toward improving this calculation
- Wish someone would tackle the comprehensive list of questions